$$= \frac{\partial}{\partial x_j}\left[\overline{u_i'\left(\frac{\partial u_i'}{\partial x_j}+\frac{\partial u_j'}{\partial x_i}\right)}\right] - \overline{\frac{\partial u_i'}{\partial x_j}\frac{\partial u_j'}{\partial x_i}} + \overline{u_i'\frac{\partial}{\partial x_i}\underbrace{\left(\frac{\partial u_j'}{\partial x_j}\right)}_{0}}$$

$$\overline{u_i'u_j'}\frac{\partial\overline{u_i}}{\partial x_j} = \overline{u_j'u_i'}\frac{\partial\overline{u_j}}{\partial x_i} = \frac{1}{2}\overline{u_i'u_j'}\left(\frac{\partial\overline{u_i}}{\partial x_j}+\frac{\partial\overline{u_j}}{\partial x_i}\right)$$

$$-\overline{\left(\frac{\partial u_i'}{\partial x_j}\right)^2} - \overline{\frac{\partial u_i'}{\partial x_j}\frac{\partial u_j'}{\partial x_i}} = -\frac{1}{2}\overline{\left(\frac{\partial u_i'}{\partial x_j}\right)^2} - \frac{1}{2}\overline{\left(\frac{\partial u_j'}{\partial x_i}\right)^2} - \overline{\frac{\partial u_i'}{\partial x_j}\frac{\partial u_j'}{\partial x_i}} = -\frac{1}{2}\overline{\left(\frac{\partial u_i'}{\partial x_j}+\frac{\partial u_j'}{\partial x_i}\right)^2}$$

を用いて

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\overline{u_i'u_i'}\right) + \overline{u_j}\frac{\partial}{\partial x_j}\left(\frac{1}{2}\overline{u_i'u_i'}\right)$$
$$= -\frac{\partial}{\partial x_j}\left(\frac{1}{\rho}\overline{u_j'p'} + \frac{1}{2}\overline{u_i'u_i'u_j'} - 2\nu\overline{u_i's_{ij}'}\right) - \overline{u_i'u_j'}\,\overline{s_{ij}} - 2\nu\overline{s_{ij}'s_{ij}'} \qquad (5.42)$$

となる．ただし，$\overline{s_{ij}}$, s_{ij}' は次のような定義である．

$$\overline{s_{ij}} \equiv \frac{1}{2}\left(\frac{\partial\overline{u_i}}{\partial x_j}+\frac{\partial\overline{u_j}}{\partial x_i}\right),\quad s_{ij}' \equiv \frac{1}{2}\left(\frac{\partial u_i'}{\partial x_j}+\frac{\partial u_j'}{\partial x_i}\right)$$

熱の分散 $\overline{\theta'^2}$ を支配する式

(5.34) に $2\theta'$ をかけて平均をとることによって求められる．

$$\frac{\partial\overline{\theta'^2}}{\partial t} + \overline{u_j}\frac{\partial\overline{\theta'^2}}{\partial x_j} = -\frac{\partial}{\partial x_j}\left(\overline{\theta'^2u_j'} - \nu_\theta\frac{\partial\overline{\theta'^2}}{\partial x_j}\right) - 2\overline{\theta'u_j'}\frac{\partial\overline{\theta}}{\partial x_j} - 2\nu_\theta\overline{\frac{\partial\theta'}{\partial x_j}\frac{\partial\theta'}{\partial x_j}}. \quad (5.43)$$

乱れの運動量フラックスを支配する式

(5.33)$\times u_k'$ と，その i と k を置換した式を加え，全体の平均をとることにより求められる．((5.41) はこの式で $k=i$ の縮約をとったものである．)

$$\frac{\partial\overline{u_i'u_k'}}{\partial t} + \overline{u_j}\frac{\partial\overline{u_i'u_k'}}{\partial x_j} = -\overline{u_i'u_j'}\frac{\partial\overline{u_k}}{\partial x_j} - \overline{u_k'u_j'}\frac{\partial\overline{u_i}}{\partial x_j} - \frac{\partial\overline{u_i'u_k'u_j'}}{\partial x_j}$$
$$-\frac{1}{\rho}\left[\frac{\partial\overline{p'u_k'}}{\partial x_i} + \frac{\partial\overline{p'u_i'}}{\partial x_k} - \overline{p'\left(\frac{\partial u_i'}{\partial x_k}+\frac{\partial u_k'}{\partial x_i}\right)}\right] + \nu\frac{\partial^2\overline{u_i'u_k'}}{\partial x_j^2} - 2\nu\overline{\frac{\partial u_i'}{\partial x_j}\frac{\partial u_k'}{\partial x_j}}. \quad (5.44)$$